Ground states and doubly excited resonance states of H$^-$ embedded in dense quantum plasmas

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Abstract
We have made an investigation on the ground states and the 2s$^2\,^1$S$^e$ resonance states of H$^-$ in dense quantum plasmas. Exponential-cosine-screened Coulomb potentials (ECSCP) are used to represent the effective potential for a test charge in dense quantum plasmas. Ground-state energies and wavefunctions are determined within the framework of Ritz’s variational principle by employing highly correlated wavefunctions to take into account the correlation effect of the charged particles. Ground-state energies are shown to converge with the increase of terms in the wavefunctions. We also report various expectation values of the coordinates of electrons in H$^-$.

Resonance energies and widths for the doubly excited H$^-$ for various values of the screening parameter are determined using the stabilization method by calculating the density of the resonance states. Results for resonance energies and widths are reported for the screening parameter in the range 0.0–0.15. Such a calculation for H$^-$ is reported for the first time in the literature.

(Some figures in this article are in colour only in the electronic version)

1. Introduction
An electron in an atom or ion, placed in vacuum, is known to interact with the nucleus via the Coulomb potential. When the atom is placed in an external environment, this potential changes and as a result the energy levels and other properties of the atom change considerably. This change in the pure Coulomb potential depends on the nature of interaction of the atom with the external environment and can be approximated by model potential. As a notable example, the potential of a test charge in a weakly coupled plasma can be modelled by the Debye–Hückel model or screened Coulomb potential (SCP) [1]:

$$V(r) = \frac{1}{r} e^{-\lambda r} \cos(\lambda r) \text{ (in au)},$$

(1)

The quantity $\lambda$, known as the screening parameter, is related to the plasma frequency $\omega_p$ by the relation $\lambda = \frac{\omega_p}{v_T}$, where $v_T$ and $\omega_p$ are the thermal velocity and plasma frequency, respectively. The effect of a weakly coupled plasma on the atomic structure has been the subject of extensive studies over the last few years [2–14] (and further references therein). However, the Debye–Hückel model is not reliable to investigate the physical properties of the quantum plasmas with the increase of plasma density due to the multi-particle cooperative interactions [15]. Recently, Shukla and Eliasson [16] have shown that the effective potential of a test charge of mass $m$ in a dense quantum plasma can be represented by a modified Debye–Hückel potential or exponential-cosine-screened Coulomb potential (ECSCP):

$$V(r) = \frac{1}{r} e^{-\lambda r} \text{ (in au)},$$

(2)

rather than the SCP as given in equation (1). Here the screening parameter $\lambda$ is related to the plasma frequency $\omega_p$ by means of the relation $\lambda = \frac{\omega_p}{(\hbar \omega_p/m)^{1/2}}$, which, in turn, is related to the quantum wave number $k_q$ by the relation $\lambda = k_q/\sqrt{2}$. This finding has attracted considerable interest in the studies atomic processes in dense quantum plasmas [17–22], though ECSCP is also known to represent the potential between an ionized impurity and an electron in metal, the ionized impurity–electron potential in a semiconductor [19].
Quantum plasmas are essentially composed of electrons and ions, and characterized by a low temperature and a high number density. If a plasma is cooled to an extremely low temperature, the de Broglie wavelength of the charge carriers may be comparable to the Debye length \( D(=1/\lambda) \) of the plasma. In such situations, the ultracold plasma behaves as a Fermi gas, and the quantum-mechanical effects play a vital role in the behaviour of the collective interactions of the charged particles. So, one can say that the quantum limit is reached when the de Broglie wavelength becomes larger than the screening length \((1/\lambda)\), such that an electron inside the Debye sphere is screened by the potential outside the sphere. In quantum plasmas, the ranges of the electron number density \( n_e \) and temperature \( T \) are, respectively, known to be about \( 10^{18} - 10^{23}\text{cm}^{-3} \) and \( 10^{2} - 10^{5} \text{K} \), and the coupling plasma parameter \( \Gamma > 1 \) [17]. The existence of a quantum plasma is quite common in different areas, such as, in micro- and nano-electronic devices, in dense astronomical systems (the white dwarfs and the neutron stars), in laser-produced plasmas, in nonlinear optics, etc [23]. Though the \( \lambda \) in equations (1) and (2) carry different physical attributes, yet form a mathematical point of view we can see that for weakly coupled plasmas, the cos \((\lambda r)\) term in equation (2) is absent, and so it becomes SCP as in equation (1). Due to the presence of the cos term, ECSCP exhibits a stronger screening effect than SCP, and this makes the properties of an atom in a dense quantum plasma different from that in a weakly coupled plasma.

For a one-electron atom interacting with ECSCP, a number of investigations with various degrees of sophistication have been reported in the literature [24–42]. For two-electron systems, to the best of our knowledge, no such investigation is available in the literature, except the recent results on helium (He) by Ghoshal and Ho [19, 20]. But the study of two-electron systems is important because it involves the correlation of the electrons. Among the two-electron systems with two charged particles, it is obviously the natural choice. But in an environment where the interaction of the charged particles is screened, H\(^{-}\) behaves differently from the other elements of the two-electron series, as its nuclear charge is unity. For elements other than H\(^{-}\) of the two-electron series, the electron–nucleus attraction dominates over the electron–electron repulsion, and with the increase of the screening effect, the electron–electron repulsion drops more rapidly than the electron–nucleus attraction. But for H\(^{-}\), both the attraction and repulsion behave alike with the increase of the screening effect. This fact has important consequences on the autoionization of the atom. A comprehensive description of a number of interesting features which are not shared by other two-electron systems is given in [43]. Furthermore, H\(^{-}\) is important in the study of our atmosphere (particularly, the ionosphere’s D-layer), of the atmosphere of the sun and other stars, and is also used as the initial species for acceleration in particle physics accelerators and plasma mechanics [43, 44].

In this paper, we focus our attention on the ground and the \( 2s^2 \, ^1S^\circ \) resonance states of H\(^{-}\) in dense quantum plasmas. Of course, it is a challenging task to determine the doubly excited states of a two-electron system and the problem still attracts the serious attention of atomic and molecular physicists [11–13, 47–53]. We design this paper as follows. Describing the underlying theory and calculations of our investigation in section 2 we present and discuss our computed results in section 3. Finally, in section 4 we give our concluding remarks.

Atomic units (au) are used throughout the present work, and all calculations are performed in quadruple precision (32 significant figures) on IBM-AMD workstations in the UNIX environment.

2. Theory and calculations

The non-relativistic Hamiltonian of H\(^{-}\) in a dense quantum plasma, characterized by ECSCP with the screening parameter \( \lambda \), is given by

\[
H = -\frac{1}{2} \nabla_1^2 - \frac{1}{2} \nabla_2^2 - \left[ \frac{e^{-\lambda r_1}}{r_1} \cos(\lambda r_1) + \frac{e^{-\lambda r_2}}{r_2} \cos(\lambda r_2) \right] + e^{-\lambda r_{12}} \cos(\lambda r_{12}),
\]

where \( r_1 \) and \( r_2 \) are the coordinates of the two electrons relative to the nucleus (assumed to be at rest), and \( r_{12} \) is their relative distance. The screening parameter \( \lambda \) determines the effect of screening. With the increase of \( \lambda \) the screening becomes stronger.

In order to determine \(^1S^\circ\) states of the H\(^{-}\) atom in dense quantum plasmas, we use the wavefunction

\[
\Psi(r_1, r_2) = \sum_{i=1}^{N} C_i \psi_i = \sum_{i=1}^{N} C_i (1 + P_12) \alpha r_{12}^{\alpha} \prod_{1 \leq l < n \leq N} r_{li}^{\alpha},
\]

where \( \alpha \) is a nonlinear variational parameter, \( C_i(i = 1, 2, 3, \ldots, N) \) are linear expansion coefficients, \( \alpha \) is a scaling constant to be discussed later, and \( P_12 \) is an exchange operator such that \( P_12 f(r_1, r_2) = f(r_2, r_1) \) for an arbitrary function \( f \). We expand this wavefunction by raising the powers \( r_1, r_2 \) and \( r_{12} \) in such a way that the terms corresponding to \( l_1 + m_1 + n_1 = \omega = 0 \text{ (N = 1)} \) come first, then \( \omega \leq 1 \text{ (N = 3)} \), \( \omega \leq 2 \text{ (N = 7)} \), and so on. To determine the ground-state energies of H\(^{-}\), we first set \( \alpha = 1 \). The wavefunction is then used in Ritz’s variational principle [45] to obtain ground-state energies and wavefunctions. The matrix elements, \( H_{ij} = \langle \psi_j | H | \psi_i \rangle \), \( S_{ij} = \langle \psi_i | S | \psi_j \rangle \), required for the execution of Ritz’s variational principle [46], are obtained in closed analytic forms for this wavefunction. The nonlinear variational parameter \( \alpha \) is optimized using the Monte Carlo optimization technique. We select randomly the variational parameters \( \alpha \). The energy eigenvalues are calculated for all of these values of the variational parameter until the fully optimized results are obtained for the wavefunction.

During optimization of the variational parameter \( \alpha \), it has been found that the variation of energy with variation of \( \alpha \) is appreciable for smaller number of terms, such as \( \omega \leq 1, 2, 3, \ldots, 8 \), in the wavefunction. But with the increase of terms in the wavefunction, the values of the parameter \( \alpha \) in a wide range produce nearly the same energy; rather the increase of the terms brings about the convergence in the energy value. This indicates that as the wavefunction is expanded throughout
the configuration space it becomes very close to the exact wavefunction. In other words, this wavefunction is a nice approximation of the exact one. However, we first optimize the parameter $A$ using 252 number of terms ($\omega \leq 12$), then expand the wavefunction by letting $\omega \leq 13, 14, 16, \ldots$ with out changing $A$ still the energy eigenvalue converges at the desired accuracy.

To determine the ground-state energies and wavefunctions of H we also use Ritz’s variational principle and employ the wavefunction,

$$
\Psi_1(r) = \sum_i C_i \psi_i = \sum_i C_i e^{-B_ir_i/l_i}, \quad l_i = 0, 1, 2, \ldots
$$

(5)

where $B_i$'s are nonlinear variational parameters, and $r$ denotes the coordinates of the electron relative to the nucleus.

In order to determine the doubly excited resonance states, we follow the stabilization method similar to Ho and coworkers [47, 48]. This method is a slight generalization of the stabilization method proposed by Mandelshtam et al [49]. The method has essentially two components: construction of the stabilization diagram and calculation of the density of resonance states. In order to construct the stabilization diagram, we first diagonalize the Hamiltonian (3) using the wavefunction (4) with different values of $\alpha$, from which we obtain the energy levels $E(\alpha)$ for a particular value of $\lambda$. We then construct stabilization diagrams (as shown in figures 1(a), 2(a) and 3(a)) by plotting $E(\alpha)$ as a function of $\alpha$. If a stabilized or slowly decreasing energy level appears in the stabilization plateau at some energy $E$, then it indicates the existence of a resonance state at the corresponding energy. The details of successful applications of this simple and powerful method are available in the various works of Ho and coworkers [47] (and further references therein). The scaling parameter $\alpha$ in the wavefunction (4) acts as the reciprocal range of a ‘soft’ wall [47].

Next we calculate the density of resonance states for a single energy level by using the formula

$$
\rho_n(E) = \left[ \frac{E_n(\alpha_{i+1}) - E_n(\alpha_{i-1})}{\alpha_{i+1} - \alpha_{i-1}} \right]^{-1} \frac{\delta}{\delta E_{n(\alpha_i)} = E},
$$

(6)

where the index $i$ is the $i$th value for $\alpha$, and the index $n$ is for the $n$th resonance. Now in order to extract the resonance energy $E_r$ and the resonance width $\Gamma$ we fit $\rho_n(E)$ to the following Lorentzian form:

$$
\rho_n(E) = y_0 + \Delta \pi \frac{l^2}{(E - E_r)^2 + \left(\frac{l}{\Gamma}\right)^2},
$$

(7)

where $y_0$ is the baseline offset, $\Delta$ is the total area under the curve from the baseline, $E_r$ is the centre of the peak, and $\Gamma$ denotes the full width of the peak of the curve at half height. This Lorentzian form yields the resonance energy $E_r$ and the resonance width $\Gamma$. 

![Figure 1](image1.png)

**Figure 1.** Ground-state energies of H and H$^-$ in ECSCP for different values of the screening parameter.

![Figure 2](image2.png)

**Figure 2.** (a) Stabilization plots of the 2s$^2$1S$^e$ states of H$^-$ in ECSCP for $\lambda = 0.0$, and $\omega \leq 14 (N = 372)$. The number in the parentheses next to the solid line indicates the order of appearance of the eigenvalues. (b) Calculated density (circles) and the fitted Lorentzian (solid line) for the 12th eigenvalue corresponding to the 2s$^2$1S$^e$ state of H$^-$ in ECSCP for $\lambda = 0.0$, leading to the pure Coulomb case.
expected, because as or decreasing. We observe that ⟨langle\rangle\rangle
But what is interesting is that the way they are increasing potential energy and kinetic energy. value, it drops rapidly. This fact is reversed in the case of
follows. From the expectation values of
⟨langle\rangle\rangle
parameter.
In table 1, we present the ground-state energies of H
3.1. Ground states
In table 1, we present the ground-state energies of H\(^-\) for some values of the screening parameter ranging from 0 to 0.7. From this table, it is seen that the energy, for each value of \(\lambda\), does converge with the increase of the terms in the wavefunction. Our reported value of the ground-state energy for \(\lambda = 0\) agrees nicely with some of the most accurate results [11, 50] available in the literature. For the other \(\lambda\) values, the uncertainties of our results are within 2–3 parts in the last reported digits. From table 1 we also see that ground-state energy of H\(^-\) increases steadily with increasing \(\lambda\) and ultimately approaches the critical value (the minimum value of \(\lambda\) for which the electrons in the ion are free) around \(\lambda = 0.7\).

In table 2, we present various expectation values of the coordinates of electrons in H\(^-\) along with the ratio of potential energy to the kinetic energy. From this table we note that \(\langle 1/r_1 \rangle\), \(\langle 1/r_{12} \rangle\) decrease with increasing \(\lambda\), whereas \(\langle r_1 \rangle\), \(\langle r_{12} \rangle\) and \(\langle r_1^2 + r_{12}^2 \rangle\) increase with increasing \(\lambda\). These are quite expected, because as \(\lambda\) increases screening becomes stronger. But what is interesting is that the way they are increasing or decreasing. We observe that \(\langle r_1 \rangle\) or \(\langle r_{12} \rangle\) decreases very slowly for small values of \(\lambda\), but as \(\lambda\) approaches the critical value, it drops rapidly. This fact is reversed in the case of \(\langle r_1 \rangle\), \(\langle r_{12} \rangle\) and \(\langle r_1^2 + r_{12}^2 \rangle\). This behaviour can be explained as follows. From the expectation values of \(\langle r_1 \rangle\) and \(\langle r_{12} \rangle\), we find that the two electrons, for the most part, are located in the two opposite sides of the nucleus at almost the same distance making angles 108\(^\circ\) (for the unscreened case) to 120\(^\circ\) (for strong screening) approximately at the nucleus. As a result, for large \(\lambda\), the screening which is of exponential cosine type is mainly effective on the electron–ion attraction. Moreover, from table 2 we see that the ratio of the potential energy to the kinetic energy is 2, when there is no screening, which is in accordance with the virial theorem. But as the screening increases it gradually decreases, and ultimately tends to unity as \(\lambda\) approaches the critical value. This is because, as \(\lambda\) approaches to the critical value the total energy as well as the potential energy tends to zero, so the kinetic energy also tends to zero. Consequently, the ratio of the potential energy to the kinetic energy tends to 1.

In figure 1, the ground-state energies of H and H\(^-\) as a function of the screening parameter are shown. From this figure we see that the ground-state energy of H\(^-\) lies below that the ground-state energy of H for any \(\lambda\). With the increase in \(\lambda\), ground-state energies of both of them increase in such a way that they attain their critical values almost at the same time.

3.2. Resonance states
In figures 2(a), 3(a) and 4(a), we present the stabilization plots for \(\lambda = 0.0, 0.05\) and 0.1, respectively. The stabilization diagram, in figure 2(a), corresponding to \(\lambda = 0.0\) in the range of \(\alpha = 0.25–0.8\) is obtained by using 372 terms in the wavefunction and covering the range of \(\alpha\) with 441 points in a mesh size of 0.001 25. This stabilization plot shows the stabilization character near \(E \approx -0.15\). We calculate, using formula (6), the density of resonance states for the individual energy levels in the range 0.25–0.8, with one energy level at a time. The calculated density of resonance states from the single energy eigenvalue is then fitted to equation (7),

<table>
<thead>
<tr>
<th>(\lambda)</th>
<th>(-E(\omega \leq 0.13))</th>
<th>(-E(\omega \leq 0.14))</th>
<th>(-E(\omega \leq 0.15))</th>
<th>(-E(\omega \leq 0.16))</th>
<th>(-E(\omega \leq 0.17))</th>
</tr>
</thead>
<tbody>
<tr>
<td>N = 252</td>
<td>N = 308</td>
<td>N = 372</td>
<td>N = 444</td>
<td>N = 525</td>
<td></td>
</tr>
<tr>
<td>0.00</td>
<td>0.527 750 98</td>
<td>0.527 751 00</td>
<td>0.527 751 01</td>
<td>0.527 751 01</td>
<td>0.527 751 01</td>
</tr>
<tr>
<td>0.05</td>
<td>0.477 720 54</td>
<td>0.477 720 55</td>
<td>0.477 720 56</td>
<td>0.477 720 56</td>
<td>0.477 720 56</td>
</tr>
<tr>
<td>0.10</td>
<td>0.427 679 21</td>
<td>0.247 679 22</td>
<td>0.247 679 22</td>
<td>0.247 679 22</td>
<td>0.247 679 22</td>
</tr>
<tr>
<td>0.15</td>
<td>0.377 962 75</td>
<td>0.377 962 76</td>
<td>0.377 962 76</td>
<td>0.377 962 76</td>
<td>0.377 962 76</td>
</tr>
<tr>
<td>0.20</td>
<td>0.329 083 66</td>
<td>0.329 083 69</td>
<td>0.329 083 70</td>
<td>0.329 083 70</td>
<td>0.329 083 70</td>
</tr>
<tr>
<td>0.25</td>
<td>0.281 599 11</td>
<td>0.281 599 14</td>
<td>0.281 599 16</td>
<td>0.281 599 17</td>
<td>0.281 599 17</td>
</tr>
<tr>
<td>0.50</td>
<td>0.082 890 1</td>
<td>0.082 890 5</td>
<td>0.082 890</td>
<td>0.082 890</td>
<td>0.082 891</td>
</tr>
<tr>
<td>0.70</td>
<td>0.001 12</td>
<td>0.001 12</td>
<td>0.001 12</td>
<td>0.001 12</td>
<td>0.001 13</td>
</tr>
</tbody>
</table>

Table 1. Convergence of the ground-state energies of H\(^-\) with the increase of terms in the wavefunctions for some values of the screening parameter. \(N\) denotes the number of terms in the wavefunction.

Table 2. Various expectation values associated with H\(^-\) for some values of the screening parameter \(\lambda\). P.E. and K.E., respectively, denote the potential energy and kinetic energy.

<table>
<thead>
<tr>
<th>(\lambda)</th>
<th>(\langle r_1^2 \rangle)</th>
<th>(\langle r_{12}^2 \rangle)</th>
<th>(\langle r_1 \rangle)</th>
<th>(\langle r_{12} \rangle)</th>
<th>(\langle r_1^2 + r_{12}^2 \rangle)</th>
<th>(\frac{P.E.}{K.E.})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.683 261 75</td>
<td>0.311 021 49</td>
<td>2.710 178 20</td>
<td>4.412 694 33</td>
<td>23.827 385 37</td>
<td>2.000 000 00</td>
</tr>
<tr>
<td>0.05</td>
<td>0.683 284 04</td>
<td>0.311 029 38</td>
<td>2.710 671 87</td>
<td>4.412 929 97</td>
<td>23.844 101 79</td>
<td>1.905 137 50</td>
</tr>
<tr>
<td>0.10</td>
<td>0.683 038 25</td>
<td>0.310 741 31</td>
<td>2.717 234 10</td>
<td>4.421 425 68</td>
<td>24.028 998 93</td>
<td>1.810 627 56</td>
</tr>
<tr>
<td>0.15</td>
<td>0.681 593 63</td>
<td>0.309 249 75</td>
<td>2.739 772 81</td>
<td>4.455 525 84</td>
<td>24.622 513 89</td>
<td>1.718 379 81</td>
</tr>
<tr>
<td>0.20</td>
<td>0.677 876 78</td>
<td>0.306 007 47</td>
<td>2.787 901 54</td>
<td>4.532 808 20</td>
<td>25.857 717 46</td>
<td>1.630 200 80</td>
</tr>
<tr>
<td>0.25</td>
<td>0.670 898 89</td>
<td>0.299 988 58</td>
<td>2.871 115 63</td>
<td>4.670 758 72</td>
<td>27.999 354 24</td>
<td>1.547 199 71</td>
</tr>
<tr>
<td>0.50</td>
<td>0.555 929 98</td>
<td>0.212 329 61</td>
<td>4.448 003 98</td>
<td>7.423 013 92</td>
<td>80.715 252 70</td>
<td>1.211 066 71</td>
</tr>
<tr>
<td>0.65</td>
<td>0.357 034 37</td>
<td>0.104 682 85</td>
<td>9.634 870 59</td>
<td>16.414 502 05</td>
<td>399.555 748 84</td>
<td>1.058 652 44</td>
</tr>
<tr>
<td>0.70</td>
<td>0.197 417 74</td>
<td>0.030 466 75</td>
<td>25.881 250 60</td>
<td>44.719 074 79</td>
<td>1161.832 184 51</td>
<td>1.000 298 04</td>
</tr>
</tbody>
</table>
Figure 3. (a) Stabilization plots of the 2s^2 1S_e states of H^− in ECSCP for λ = 0.05, and \( \omega \leq 15(N = 444) \). The number in the parentheses next to the solid line indicates the order of appearance of the eigenvalues. (b) Calculated density (circles) and the fitted Lorentzian (solid line) for the 13th eigenvalue corresponding to the 2s^2 1S_e state of H^− in ECSCP for λ = 0.05.

and the one that gives the best fit (with the least \( \chi^2 \)) to the Lorentzian form is considered as the desired result for that particular resonance. Figure 2(b) shows the fitting of the density of resonance states for the 12th eigenvalue of the stabilization diagram (Figure 2(a)). From the fit, we obtain the resonance energy \( E_r = -0.14876 \) and the corresponding width as \( \Gamma = 0.001732 \). These results for the unscreened case (λ = 0.0) are in nice agreement with those reported by Kar and Ho [12] using the stabilization method and by Chakraborty and Ho [13] using the complex coordinate rotation method. This fact also ensures the accuracy of our results for other values of λ.

In order to test the convergency of our calculation, we construct the stabilization plots for \( \lambda = 0.05 \) using three different expansions of the wavefunction (4), namely, \( \omega \leq 12(N = 252), \omega \leq 14(N = 372) \) and \( \omega \leq 15(N = 444) \), in the range \( 0.25 \leq \alpha \leq 0.7 \). This range of \( \alpha \) is covered by 451, 361, 451 points with the mesh sizes 0.001, 0.00125 and 0.001, respectively. The results of resonance parameters for these three expansions of the wavefunction (4) are shown in table 3.

Table 3. Convergence of the resonance energy (\( E_r \)) and width (\( \Gamma \)) using \( \omega \leq 12(N = 252), \omega \leq 14(N = 372) \) and \( \omega \leq 15(N = 444) \) for the screening parameter \( \lambda = 0.05 \). N denotes the number of terms in the wavefunction.

<table>
<thead>
<tr>
<th>N</th>
<th>( E_r )</th>
<th>( \Gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>0.097693</td>
<td>0.001765</td>
</tr>
<tr>
<td>14</td>
<td>0.097699</td>
<td>0.001764</td>
</tr>
<tr>
<td>15</td>
<td>0.097702</td>
<td>0.001764</td>
</tr>
</tbody>
</table>

From this table we see that our results of resonance energy and resonance width converge with the increase of terms in the wavefunction. Figures 3(a) and 4(a) are the stabilization plots for \( \lambda = 0.05 \) and \( \lambda = 0.1 \), whereas figures 3(b) and 4(b) are the corresponding Lorentzian fittings. These stabilization plots are made in the range of \( 0.25 \leq \alpha \leq 0.7 \) and \( 0.25 \leq \alpha \leq 0.8 \) with mesh sizes 0.00125 and 0.001, respectively. It should be mentioned here that when density of the resonance states \( \rho_n(E) \) is fitted to the Lorentzian form, it is found that the value of \( \chi^2 \) for each fitting is much less than 0.1.

Our present resonance energies associated with the \( N = 2 \) threshold of the hydrogen atom along with the H(2s) threshold
energies for different values of \( \lambda \) are shown in figure 5(a). The corresponding widths are shown in figure 5(b). From figure 5(a), it is seen that the resonance energy gradually increases with the increasing value of \( \lambda \) and ultimately becomes very close to the \( \text{H}(2s) \) energy at \( \lambda = 0.15 \). It is also evident from figure 5(a) that, beyond \( \lambda = 0.15 \), Feshbach-type resonances lying below the \( \text{H}(2s) \) threshold, if there be any, would be located very near the \( \text{H}(2s) \) threshold. However, such calculations would need more extensive basis sets, and are outside the scope of our present investigation.

Moreover, figure 5(b) shows that the resonance width \( \Gamma \), first, increases very slightly with the increasing value of \( \lambda \), and then decreases rapidly. This is a particular aspect of \( \text{H}^- \) interacting ECSCP. Such behaviour is not found in the case of \( \text{He} \) in ECSCP [20], or even in the case of \( \text{H}^- \) interacting SCP (as is seen from table 4). It seems that this is due to the combined effect of the oscillatory nature of ECSCP and the equality of the attractive and repulsive forces in \( \text{H}^- \).

We know that the mixing of a configuration belonging to a discrete spectrum (the so-called closed space \( Q\Psi \) in the Feshbach projection formulism) with a continuous spectrum (the so-called open space \( P\Psi \) in the Feshbach projection formulism) configuration gives rise to the phenomenon of autoionization [51] by ejecting an electron. The resonance width is given by \( \Gamma = 2\pi|V_E|^2 \), where \(|V_E|^2\) is an index of the strength of the configuration interaction [52]. \(|V_E|^2\) is characterized by the product of two different factors; one is determined by the configuration interaction within the closed space of the doubly excited two-electron \( \text{H}^- \), whereas the other (the open space) is determined by the details of the field at large distance from the \( \text{H}(1s) \) ground state, the state to which the doubly excited \( 2s^21S^e \) resonance autoionized. So a rigorous explanation of the behaviour of resonance width needs an extensive study of these two factors. It is of interest to investigate this issue further in the future.

In table 4, we compare our present results in a dense quantum plasma with those of Kar and Ho in a weakly coupled plasma [12], and also the precision calculation of Ho et al [53] for the unscreened case. Nice agreement of our result with the precision calculation of Ho et al [53] for \( \lambda = 0 \) once again establishes the accuracy of our calculation. We estimate the total uncertainties for the present calculations are about \( 2 \times 10^{-5} \) au, each for the resonance energy and the width. It is to be noted that we have used ECSCP to take into account the screening effect of dense quantum plasmas, which presumably accounts for increased density or decreased temperature as required to reach the quantum limit. On the other hand, SCP has been used to represent the screening effect of weakly coupled plasmas. As ECSCP (or equivalently a dense quantum plasma) shows stronger screening effect than SCP (or equivalently a weakly coupled plasma), the energy levels of an atom or ion in a dense quantum plasma are shifted to the higher positions with respect to those in a weakly coupled plasma. This is evident from table 4. Also the critical value of

### Table 4

The resonance energies \( (E_r) \) and widths \( (\Gamma) \) of \( \text{H}^- \) in a dense quantum plasma and a weakly coupled plasma for various values of the screening parameter \( \lambda \).

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>Present results in a dense quantum plasma</th>
<th>Results of Kar and Ho [12] in a weakly coupled plasma</th>
<th>Precision calculation of Ho et al [53]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.14876 0.001732</td>
<td>0.14876 0.001733</td>
<td>0.14877 0.001733</td>
</tr>
<tr>
<td>0.01</td>
<td>0.13875 0.001733</td>
<td>0.13885 0.001730</td>
<td>0.13885 0.001730</td>
</tr>
<tr>
<td>0.02</td>
<td>0.12865 0.001738</td>
<td>0.12917 0.001721</td>
<td>0.12917 0.001721</td>
</tr>
<tr>
<td>0.05</td>
<td>0.09770 0.001663</td>
<td>0.10182 0.001638</td>
<td>0.10182 0.001638</td>
</tr>
<tr>
<td>0.1</td>
<td>0.04686 0.001309</td>
<td>0.06311 0.001309</td>
<td>0.06311 0.001309</td>
</tr>
</tbody>
</table>

Figure 5. (a) The \( 2s^21S^e \) resonance energy \( E_r \) for different values of the screening parameter \( \lambda \). The dashed line denotes the \( \text{H}(2s) \) threshold energy. (b) Resonance width, \( \Gamma \), corresponding to the resonance energy in (a) for different values of the screening parameter \( \lambda \).
the screening parameter in a weakly coupled plasma is larger than that in a dense quantum plasma. Furthermore, here we calculate resonances lying below the H(2s) threshold. As we have already mentioned in the last paragraph, a detailed behaviour of the resonance width with the increasing screening parameter larger than those reported here would be of interest in future investigation.

4. Conclusions

In this paper, we have made an investigation, for the first time, on the ground and doubly excited resonance states of H\textsuperscript{−} embedded in dense quantum plasmas. The plasma screening effect has been taken into account by the exponential-cosine-screened Coulomb potential. Highly correlated Hylleraas-type wavefunctions have been used to take into account the electron–electron correlation effect. Our results of ground and resonance states for the unscreened case agree nicely with the electron–electron correlation effect. Our results of ground and doubly excited resonance states have been used to take into account the plasma screening effect has been taken into account by the exponential-cosine-screened Coulomb potential.

Conclusions

In future investigation.

Acknowledgments

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