COHERENT LIGHT FIELD CONVERSION BASED ON FOUR-LEVEL ATOMIC SYSTEM

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Lab.330 | Group meeting
Coherent light field conversion based on four-level atomic system

- **Introduction**
  - Coherent light field conversion
  - Clebsch-Gordan coefficients mismatch in atomic system
    - Conversion loss and protection
    - EIT bandwidth interpretation

- **Theoretical model**
  - Initial conditions: An initial spin wave coherence
  - Optical Bloch equations
    - Equation of motion for spin wave coherence
  - Maxwell-Schrödinger equations
    - Laplace transform
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- **Discussion**
  - Bandwidth narrowing and boarding by CG-coefficients mismatch

- **Summary**
Coherent light field conversion

Clebsch-Gordan coefficients mismatch in atomic system

Role of degenerate Zeeman states in the storage and retrieval of light pulses

Pei-Chen Guan, Yong-Fan Chen, and Ite A. PHYSICAL REVIEW A 75, 013812 2007

The ratio of the retrieved and stored energy densities

\[
\frac{u_{\text{out}}}{u_{\text{in}}} = \frac{\left[ \sum_j R_j^{(r)} R_j^{(w)} \right]^2}{\sum_j [R_j^{(r)}]^2 \sum_j [R_j^{(w)}]^2} \leq 1.
\]

\[ R_j = \frac{a_{p,j}}{a_{c,j}} \]

For single Zeeman state

retrieved polarization. Nevertheless, the symmetry of the Clebsch-Gordan coefficients is not the necessary condition for avoiding energy loss. With the detailed analysis in this work, it is now clear that one can manipulate the retrieved polarization without energy loss as long as all population accumulates in a single Zeeman state.
Coherent light field conversion

Coherent light field conversion based on four-level atomic system

Double-Λ light field conversion system
Coherent light field conversion

Coherent light field conversion based on four-level atomic system

Fig. Conversion pulse with different CG-coefficient. \( \alpha_1 = 400 \); \( \alpha_2 = |c_{sp}|^2 400 \);
\( \Omega_1 = 4.8 \Gamma \); \( \Omega_2 = 4.8 \Gamma |c_{sp}| \);

\[
|c_{sp}|^2 = \left| \frac{c_s}{c_p} \right|^2
\]

\[
E_c = \frac{1}{|c_{sp}|^2} \frac{\int_{-\infty}^{\infty} dt \Omega_s (L, t)^2}{\int_{-\infty}^{\infty} dt \Omega_p (L, t)^2}
\]
Coherent light field conversion

Clebsch-Gordan coefficients mismatch in atomic system

EIT bandwidth: $\Delta \omega_{EIT} = \sqrt{\ln 2} \frac{|\Omega c|^2}{\sqrt{\alpha_1 \Gamma^2}} \Rightarrow \Delta \omega_{EIT} T_d \propto \sqrt[\alpha_1]$
Coherent light field conversion

Clebsch-Gordan coefficients mismatch in atomic system

Following the EIT role...

\[ \Delta \omega_{EIT} = \sqrt{\ln2 \frac{|\Omega_1|^2}{\sqrt{\alpha_1 \Gamma^2}}} \Rightarrow \Delta \omega_{EIT} \propto \frac{1}{\sqrt{\alpha_1}} \]
Coherent light field conversion

Clebsch-Gordan coefficients mismatch in atomic system

Following the EIT role...

\[ \Delta \omega_{EIT} = \sqrt{\ln 2} \frac{|\Omega_1|^2}{\sqrt{\alpha_1 \Gamma^2}} \Rightarrow \Delta \omega_{EIT} \propto \frac{1}{\sqrt{\alpha_1}} \]

**HOWEVER…**
Coherent light field conversion

Coherent light field conversion based on four-level atomic system

**Fig.** Conversion pulse with different CG-coefficient. $\alpha_1 = 400; \alpha_2 = |c_{sp}|^2 400; \Omega_1 = \Omega_2 = 4.8 \Gamma$.
Coherent light field conversion

Coherent light field conversion based on four-level atomic system

Fig. Conversion pulse with different CG-coefficient. $\alpha_1 = 400; \alpha_2 = |c_{sp}|^2400; \Omega_1 = \Omega_2 = 4.8\Gamma$;
Coherent light field conversion
Coherent light field conversion based on four-level atomic system

Fig. Conversion efficiency with different CG-coefficient. Two curves are almost same even with different coupling Rabi frequency.
Two Questions:

1. The conversion efficiency \( \frac{E_{\text{conv}}}{E_{\text{slowlight}}} \) is independent on coupling Rabi frequency for the conversion channel. **WHY?**

2. The conversion efficiency is dependent on CG-coefficient ratio (optical depth ratio) for conversion channel only. **WHY?**
Theoretical model

Initial conditions: An initial spin wave coherence

Maxwell-Schrodinger equation
\[
\left( \frac{\partial}{\partial z} + \frac{1}{c} \frac{\partial}{\partial t} \right) \Omega_s(z, t) = \frac{i \alpha_2 \Gamma}{2L} \sigma_{31}(z, t)
\]

Optical Bloch equations
\[
\frac{\partial}{\partial t} \sigma_{31}(z, t) = \frac{i}{2} \Omega_2 \sigma_{21}(z, t) + \frac{i}{2} \Omega_s(z, t) - \frac{\Gamma}{2} \sigma_{31}(z, t)
\]

Initial ground state coherence
\[
\sigma_{21}(z, 0) = \sigma_s(z)
\]

Signal field: \( \Omega_s(z, t) \)

Coupling field: \( \Omega_2 \)

Atomic medium

Maxwell-Schrodinger equation

Optical Bloch equations
Theoretical model

Equation of motion for ground state coherence

\[
\left( \frac{\partial^2}{\partial^2 t} + \frac{\Gamma}{2} \frac{\partial}{\partial t} + \frac{|\Omega_2|^2}{4} \right) \sigma_{21} = -\frac{1}{4} \Omega_2^* \Omega_s(z, t)
\]

Damping \hspace{1cm} \text{Spring} \hspace{1cm} \text{Driving force}

\[
\sigma_{21}^c(z, t) = e^{-\frac{\Gamma t}{4}} (A e^{i\delta t} + B e^{-i\delta t});
\]

\[
\sigma_{21}^p(z, t) = -\frac{1}{4} \Omega_2^* \int_0^t \Omega_s(z, \xi) e^{-\frac{\Gamma (t-\xi)}{4}} \text{sinc}(\delta (t - \xi))(t - \xi) d\xi
\]

\[
= -\frac{1}{4} \Omega_2^* \left[ \Omega_s(z, t) * e^{-\Gamma t/4} t \text{sinc} (\delta t) \right](t)
\]

\[
\sigma_{21}^c(z, t) = e^{-\frac{\Gamma t}{4}} \left( \frac{\Gamma t}{4} \text{sinc}(\delta t) + \cos(\delta t) \right) \sigma_s(z)
\]

Initial conditions:
\[
\begin{aligned}
\sigma_{21}(z, 0) &= \sigma_s(z) \\
\sigma_{31}(z, 0) &= 0
\end{aligned}
\]

Where \( \delta = \frac{1}{2} \sqrt{|\Omega_2|^2 - \left(\frac{\Gamma}{2}\right)^2} \)

\[
\sigma_{21}(z, t) = \sigma_{21}^c(z, t) + \sigma_{21}^p(z, t) = \chi(t) \sigma_s(z) + \sigma_{21}^p(z, t)
\]

For coupling 2 field \( \Omega_2 \) equal to 0, we find:

\[
\sigma_{21}(z, t) = \sigma_s(z) \bigg|_{\Omega_2=0} \quad \text{(Light storage)}
\]
Theoretical model
Maxwell-Schrodinger equation - Laplace transform

\[
\left( \frac{\partial}{\partial z} + \frac{1}{c} \frac{\partial}{\partial t} \right) \Omega_s(z, t) = \frac{i\alpha_2}{2L} \frac{\partial}{\partial t} \sigma_{31}(z, t)
\]

Optical depth for conversion channel

\[
\left( \frac{\partial}{\partial z} + \frac{s}{c} \right) R_s(z, s) = \frac{\alpha_2}{\Omega_2^* L} \left( \frac{\partial}{\partial t} \chi(t) \sigma_s(z) - \frac{\partial}{\partial t} \left( \frac{1}{4} \Omega_2^* \left[ \Omega_s(z, t) * e^{-\frac{rt}{4} \text{sinc}(\delta t)} \right] (t) \right) \right)
\]

Laplace transform

Where \( \mathcal{L}_S \left[ \frac{\partial}{\partial t} \chi(t) \right] = -\frac{|\Omega_2|^2}{4} \mathcal{L}_S \left[ e^{-\frac{rt}{4} \text{sinc}(\delta t)} \right] = -\frac{|\Omega_2|^2/4}{s(s+\Gamma/2)+|\Omega_2|^2/4} \)
Theoretical model

Maxwell-Schrodinger equation - Initial condition: Gaussian distribution

$$\Rightarrow \left( \frac{\partial}{\partial z} + f(s) \right) \mathcal{R}_s(z, s) = A_s(s)\sigma_s(z)$$

$$\mathcal{R}_s \left( z = -\frac{L}{2}, s \right) = 0$$

$$\mathcal{R}_s(z, s) = A_s(s)[\sigma_s(z) \ast \exp[-f(s)z]](z)$$

EXAMPLE

In our case: $$\sigma_s(z) = \sigma_0 \exp \left[ -\left( \frac{z}{a_0} \right)^2 \right]$$

$$\mathcal{R}_s(z, s) = A_s(s)\sigma_0 \int_{-\frac{L}{2}}^{\frac{L}{2}} \exp \left[ -\frac{z^2}{a_0^2} + f(s)(z' - z) \right] dz'$$

$$= A_s(s)\sigma_0 a_0 \sqrt{\pi} \exp \left[ f(s) \left( \frac{a_0}{2} \right)^2 f(s) - z \right]$$

$$A_s(s) = \frac{\alpha_2 \Gamma}{\Omega_2^* L} \mathcal{L}_s \left[ \frac{\partial}{\partial t} \chi(t) \right]$$

$$f_s(s) = \frac{s}{c} + \frac{\alpha_2 \Gamma}{\Omega_2^* L} \mathcal{L}_s \left[ e^{-\frac{\Gamma t}{4}} \text{sinc}(\delta t) \right]$$

In our case: $$\sigma_s(z) = \sigma_0 \exp \left[ -\left( \frac{z}{a_0} \right)^2 \right]$$

$$\mathcal{R}_s(z, s) = A_s(s)\sigma_0 \int_{-\frac{L}{2}}^{\frac{L}{2}} \exp \left[ -\frac{z^2}{a_0^2} + f(s)(z' - z) \right] dz'$$

$$= A_s(s)\sigma_0 a_0 \sqrt{\pi} \exp \left[ f(s) \left( \frac{a_0}{2} \right)^2 f(s) - z \right]$$

$$a_0 < L$$
Theoretical model
Maxwell-Schrodinger equation - Initial condition: Gaussian distribution

\[ \mathcal{R}_s(z, s) = A_s(s)\sigma_0 a_0 \sqrt{\pi} \exp \left[ \frac{1}{4} f(s)(a_0^2 f(s) - 4z) \right] \]

Amplitude grows of field

Long time limit

\[ A_s(s) = \frac{\alpha_2 \Gamma}{\Omega_2^* L} \left( -1 + \frac{2\Gamma s}{|\Omega_2|^2} + \ldots \right) \]

Pulse profile

\[ \frac{1}{4} f(s)(a_0^2 f(s) - 4z) = -\frac{z}{v_g} s + \frac{a_0^2}{v_g^2} \left( 1 + \frac{8Lz}{\alpha_2 a_0^2} \right) s^2 \]

Time delay

\[ \mathcal{L}_s^{-1}[\mathcal{R}_s(z, s)] = \Omega_s(z, t) = -\frac{\sigma_0 \Omega_2}{\xi(z)} \exp \left[ -\left( \frac{z - v_g t}{a_0 \xi(z)} \right)^2 \right], \xi(z) = \sqrt{1 + \frac{8}{\alpha_2 \kappa^2} \frac{z}{L}} \]

Pulse width boarding
Discussion

Numerical Simulation v.s Field solution

Fig. Numerical Simulation v.s Field solution in pulse region.
Discussion

Conversion efficiency

\[
\sqrt{\frac{\pi \sigma_0^2 a_0 \Gamma}{2L}} \frac{\alpha_2}{\left(1 + \frac{(2/\kappa)^2}{\alpha_2}\right)^{1/2}}
\]

\[
E_c = \frac{1}{|c_{sp}|^2} \int_{-\infty}^{\infty} |\Omega_s(z=L/2,t)|^2 dt = \sqrt{1 + \frac{1}{\alpha_1 (2/\kappa)^2}} \frac{\alpha_2}{\sqrt{1 + \frac{1}{\alpha_2 (2/\kappa)^2}}}, \quad \text{(Coupling Rabi frequency independent!)}
\]

\[\alpha_2 = |c_{sp}|^2 \alpha_1\]

Fig. Conversion efficiency fitting

**ODs_1 = 100**

**ODs_1 = 500**
Discussion

Bandwidth of conversion pulse

Two Questions:
1. The conversion efficiency ($E_{\text{conv}}/E_{\text{slowlight}}$) is independent on coupling Rabi frequency for conversion channel. *WHY?*
2. The conversion efficiency is dependent on optical depth for conversion channel only. *WHY?*

Power spectral of conversion pulse:

$$S(\omega) = |\mathcal{F}_\omega [\Omega_s(z, t)]|^2 \propto \exp \left[ -\frac{1}{2} \left( \frac{a_0 \xi}{v_g} \right)^2 \omega^2 \right] \Rightarrow \Delta \omega_c = \sqrt{2 \ln 2} \frac{|\Omega_2|^2}{\left( \frac{\alpha_2 \Gamma \kappa}{2} \right)^2 + \alpha_2 \Gamma^2}^{1/2}$$

EIT-bandwidth: $\Delta \omega_{EIT} = \sqrt{2 \ln 2} \frac{|\Omega_2|^2}{[\alpha_2 \Gamma^2]^{1/2}}$ (half medium)

Conversion Bandwidth ratio: $R_c = \frac{\Delta \omega_{\text{conv}}(\alpha_2)}{\Delta \omega_{EIT}(\alpha_2)} = \frac{1}{\left[ \alpha_2 (\kappa/2)^2 + 1 \right]^{1/2}}$ (dependent on optical depth only)

EIT-Slow light Bandwidth ratio: $R_s = \frac{\Delta \omega_{\text{slow}}(\alpha_1)}{\Delta \omega_{EIT}(\alpha_1)} = \frac{1}{\left[ \alpha_1 (\kappa/2)^2 + 1 \right]^{1/2}}$
Discussion

Bandwidth narrowing and boarding due to CG-coefficients mismatch

![Bandwidth comparison graph]

Fig. Bandwidth comparison

- Conversion loss region
- Energy protection region

- $OD_{S_1} = 100$
- $OD_{S_1} = 500$
- $OD_{S_1} = 1000$

- $R_{c,OD_{S}} = 100, 500, 1000$
- $R_{s,OD_{S}} = 100, 500, 1000$
Discussion

Bandwidth narrowing and boarding due to CG-coefficients mismatch

**Fig. Conversion efficiency with different ODs**

Conversion loss region

Energy protection region

Conversion efficiency (Conv./EIT)

CG-coefficient $^{2}$ ratio

ODs$_{1}$ = 1000

ODs$_{1}$ = 500

ODs$_{1}$ = 100
Summary

Coherent light field conversion based on four-level atomic system

- Light field conversion
  - Ground state coherence $\rightarrow$ Light field

1. The conversion field nature is determined by the parameter of conversion channel.
2. Bandwidth of conversion field is always narrower than EIT bandwidth.

\[
R_c(\alpha_2) = \frac{\Delta \omega_{\text{conv.}}(\alpha_2)}{\Delta \omega_{\text{EIT}}(\alpha_2)} = \frac{1}{[\alpha_2(\kappa/2)^2 + 1]^{1/2}}
\]

3. $R_c(\alpha_2) > R_s(\alpha_1)$ $\Rightarrow$ Conversion loss region
   $R_c(\alpha_2) < R_s(\alpha_1)$ $\Rightarrow$ Energy protection region
   - $|c_{sp}|^2 < 1$ $\Rightarrow$ Conversion loss region
   - $|c_{sp}|^2 > 1$ $\Rightarrow$ Energy protection region

Bandwidth
Summary

Coherent light field conversion based on four-level atomic system

![Diagram of energy scheme](image)

*Fig. Energy scheme of light field conversion system in cesium D\textsubscript{1}-line.*

*Fig. Estimate conversion efficiency with mismatch CG-coefficient.*

\[ c_s = -\sqrt{1/48} \]
\[ c_p = -\sqrt{7/12} \]
THANK YOU
FOR YOUR ATTENTION